Algebra and Functions Strand

The Algebra and Functions strand is most closely associated with the representation of quantitative relationships, such as functions, equations, graphs, geometric diagrams, and verbal expressions. As students increase their mathematical knowledge and skills, they work frequently with algebraic symbols, expressions with variables, and graphical representations. It is essential that students develop an understanding of several different meanings and uses of variables through multiple representations. Everyday experiences with linear functions should aid in the development of the concepts of proportionality and the ability to discriminate between linear and nonlinear functions. Students must also learn to recognize and generate equivalent expressions, solve linear equations, and effectively use formulas.

To demonstrate achievement in this strand, students will be asked to:

- work with patterns and relationships
- represent, analyze, and generalize a variety of patterns with tables, graphs, and symbolic rules
- compare different forms of representations
- identify functions
- use algebraic expressions
- solve linear equations.

The use of mathematical models to represent and understand quantitative relationships is developed by modeling and solving contextualized problems. The analysis of change in various contexts involves tools such as graphs to analyze the nature of changes in quantities in linear relationships.

The ten California academic content standards covered by the CAHSEE Algebra and Functions strand are discussed in the following pages.

Strand **Algebra and Functions (AF)**

Standard **7AF1.1**
Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).

Constructs **Procedural Skills, Conceptual Understanding, Problem Solving**

Which of the following inequalities represents the statement, "A number, *x*, decreased by 13 is less than or equal to 39"?

A $13 - x \geq 39$

B $13 - x \leq 39$

C $x - 13 \leq 39$

D $x - 13 < 39$

M03049

Translating verbal descriptions into mathematical expressions is essential in solving real-world problems. CAHSEE test questions in this standard require students to translate between verbal descriptions and mathematical equivalents. Students should be able to use variables and appropriate operations to write or identify an expression, an equation, a system of equations or inequality to solve a problem.

Sample Test Question
The sample question requires students to translate a verbal description of an inequality into a mathematical expression. The correct answer is choice C. Students should recognize that "a number, *x,* decreased by 13" is represented as $x - 13$ and that "less than or equal to 39" is represented by ≤ 39. Putting both parts of the statement together, $x - 13 \leq 39$.

Analysis of Distractors
The distractors offer expressions that use the same values found in the problem but represent verbal expressions not given in the stem. Distractor A represents "13 decreased by a number, *x*," and also "greater than or equal to 39." Distractor B uses the correct inequality notation but, like Distractor A, represents "13 decreased by a number, *x*." Distractor D presents the appropriate expression for "a number, *x,* decreased by 13" but represents "less than 39," rather than "less than or equal to 39."

Strand	**Algebra and Functions (AF)**
Standard	**7AF1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x+5)^2$**
Constructs	**Procedural Skills, Conceptual Understanding**

If $h=3$ and $k=4$, then $\frac{hk+4}{2}-2=$

A 6

B 7

C 8

D 10

M00052

CAHSEE test questions for this standard require students to select and use the correct order of arithmetic operations in evaluating expressions (parentheses, exponents, multiplication, division, addition, subtraction). Students may also be required to evaluate expressions that include the distributive property and other basic properties of real numbers.

Sample Test Question

The sample question asks students to evaluate an expression using the correct order of operations. The correct answer is choice A. Students should first substitute 3 and 4 for *h* and *k*, then multiply *h* by *k*, add 4, divide by 2, and then subtract 2:

$$\frac{(3)(4)+4}{2}-2=\frac{12+4}{2}-2=\frac{16}{2}-2=8-2=6.$$

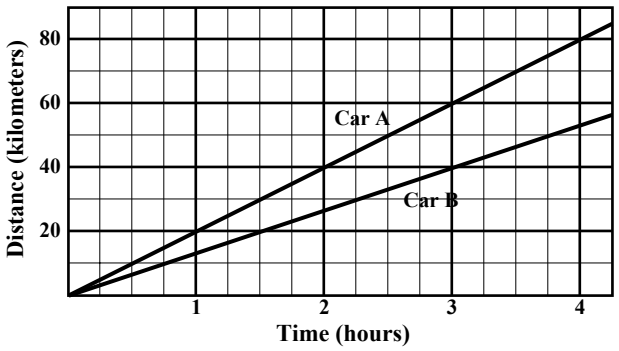
Analysis of Distractors

The distractors offer solutions that result from performing operations in improper order. Distractor B is obtained by dividing by 2 before evaluating the numerator. Distractor C is obtained by dividing the product of 3 and 4 by 2 before evaluating the numerator. Distractor D is obtained by adding 4 before multiplying 3 by 4.

Strand **Algebra and Functions (AF)**

Standard **7AF1.5**
Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

Constructs **Conceptual Understanding, Problem Solving**



After three hours of travel, Car A is about how many kilometers ahead of Car B?

A 2

B 10

C 20

D 25

M00066

CAHSEE test questions for this standard focus on either of its two main components. The first component requires the selection and execution of a graph that accurately and appropriately represents a quantitative relationship. The second component requires the interpretation and/or alternate representation of information presented in graphical form.

Sample Test Question
The sample question presents a graph showing distance traveled over time for two cars, A and B, and asks for a specific interpretation of the information shown in the graph. The correct answer is choice C. Students should recognize that distance, in kilometers, is recorded on the *y*-axis, and time, in hours, is recorded on the *x*-axis. Three hours on the time scale corresponds to 60 kilometers for Car A, and three hours corresponds to 40 kilometers for Car B. The number of kilometers that Car A is ahead of Car B after 3 hours is represented by the difference between the distance traveled by Car A and the distance traveled by Car B in the same time (60 kilometers $-$ 40 kilometers $=$ 20 kilometers).

Analysis of Distractors
The distractors represent misreadings of the graph. Distractor A gives the number of hours that Car A has traveled when it has gone 40 kilometers, the distance Car B traveled in 3 hours. Distractor B represents an error in reading the scale of the graph, assuming that the increments have a value of 10. Distractor D represents use of the approximate difference in distance at 4 hours, rather than 3.

Strand **Algebra and Functions (AF)**

Standard **7AF2.1**
Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

Constructs **Procedural Skills, Conceptual Understanding**

$x^3y^3 =$

A $9xy$

B $(xy)^6$

C $3xy$

D $xxxyyy$

M02879

This standard has four main components: the concept of positive whole-number powers as repeated multiplication, the concept of negative whole-number powers as repeated division, multiplication by the multiplicative inverse, and simplification and evaluation of expressions that include exponents. The first two components of this standard are assessed in the related Number Sense standards 2.1 and 2.3. CAHSEE test questions that assess student achievement in this standard may require students to evaluate monomial expressions. Other questions for this standard may require students to demonstrate an understanding of the multiplicative inverse.

Sample Test Question
The sample question asks students to interpret as repeated multiplication the algebraic expression for x raised to the third power multiplied by y raised to the third power. The correct answer is choice D. Students should recognize that $x^3 = x \bullet x \bullet x$ and that $y^3 = y \bullet y \bullet y$, so that $x^3y^3 = x \bullet x \bullet x \bullet y \bullet y \bullet y = xxxyyy$.

Analysis of Distractors
The distractors represent misunderstandings of the meaning of the exponents and/or the appropriate operation. Distractor A multiplies the exponents and uses the product as a coefficient. Distractor B adds the exponents, as would be appropriate for like, rather than unlike, variables. In distractor C, the value of the exponents has been moved to serve as a coefficient.

Strand **Algebra and Functions (AF)**

Standard **7AF2.2**
Multiply and divide monomials; extend the process of taking powers and extracting roots to monomials when the latter results in a monomial with an integer exponent.

Construct **Conceptual Understanding**

Simplify the expression shown below.

$$(6a^4bc)(7ab^3c)$$

A $13a^4b^3c$

B $13a^5b^4c^2$

C $42a^4b^3c$

D $42a^5b^4c^2$

M02109

CAHSEE test questions in this standard require students to multiply and divide monomials, expand powers, and find roots for monomials when the results are integer exponents. Items may also include combinations of multiplying and dividing monomials. Students should be comfortable with the rules for multiplying and dividing exponential expressions with the same base.

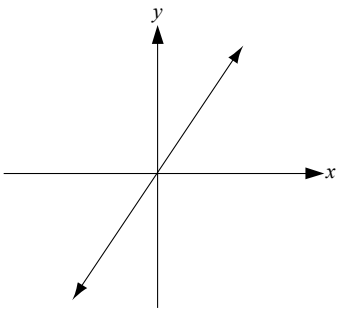
Sample Test Question
The sample question requires students to demonstrate knowledge of the rules for multiplying monomials, multiplying the numeric values, and adding the exponents. The correct answer is choice D, as $6 \times 7 = 42$ and the product of a^4 and a is a^5; the product of b and b^3 is b^4; and the product of c and c is c^2.

Analysis of Distractors
The distractors represent errors in multiplying the numeric values and/or the exponents. Distractor A represents the incorrect operation of adding, rather than multiplying, 6 and 7 and also the failure to use a as a^1, b as b^1, and c as c^1 when adding the values of the exponents. Distractor B represents the incorrect value obtained by adding 6 and 7 combined with the correct values for the exponents. Distractor C represents the correct multiplication of the numeric values but, like Distractor A, has the incorrect multiplication of the exponents a, b, and c.

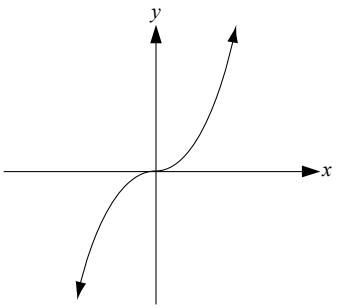
Strand	**Algebra and Functions (AF)**
Standard	**7AF3.1** **Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.**
Construct	**Conceptual Understanding**

Which of the following could be the graph of $y = x^3$?

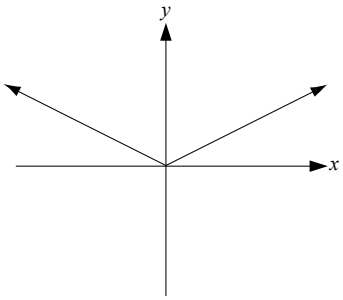
A



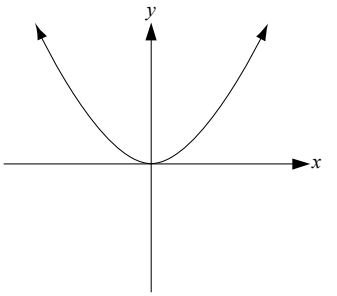
C



B



D



M02200

CAHSEE test questions in this standard require knowledge of graphing functions, as demonstrated by selecting the appropriate graph of a given function or by selecting the appropriate function for a given graph. Some questions may also require knowledge of function graphing to solve problems. As part of their foundational understanding of functions, students should be able to predict the shape of a graph based on the characteristics of the given function (e.g., linear, quadratic).

Sample Test Question

The sample question asks students to identify which graph could represent the function $y = x^3$. The correct answer is choice C. Students should understand the basic concepts underlying the problem—that cubic functions are nonlinear and that negative values for *x* correspond to negative values for *y* and positive values for *x* correspond to positive values for *y*.

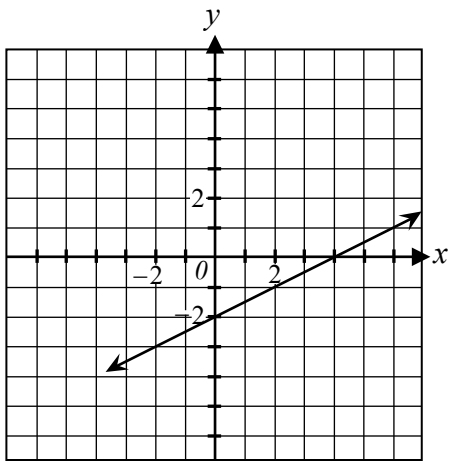
Analysis of Distractors

Distractor A is a graph of a linear, rather than nonlinear, function. Distractor B is an absolute value function. Distractor D represents a quadratic function in which negative values for *x* correspond to positive values for *y*.

Strand **Algebra and Functions (AF)**

Standard **7AF3.3**
Graph linear functions, noting that the vertical change (change in *y*-value) per unit of horizontal change (change in *x*-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.

Constructs **Procedural Skills, Conceptual Understanding**



What is the slope of the line shown in the graph above?

A –2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2

M02556

Students should understand that linear functions can model many real-world phenomena and that the rate of change in a function is shown by the slope of the graph of the function. A conceptual understanding of slope can be a key element in students' development of proportional reasoning skills. CAHSEE test questions for this standard may focus on either of its two main components. The first component is graphing linear functions on the *xy* coordinate system. The second is the identification of the slope in quantitative terms from a given linear function or the selection of a given slope from a numerical value, from a line shown on a graph, or from two pairs of coordinate points.

Sample Test Question
The sample question shows a graph of a linear function and asks students to determine the slope represented by a line that crosses the *y*-axis at the point (0, –2) and the *x*-axis at the point (4, 0). The correct answer is choice C. Students should understand slope as the change in *y* divided by the change in *x* and/or as the ratio "rise over run." In this problem, the change in the *y*-value is

obtained by subtracting –2 from 0, and the change in the *x*-value is obtained by subtracting 0 from 4, and thus $\frac{\text{Change in } y}{\text{Change in } x} = \frac{0-(-2)}{4-0} = \frac{2}{4} = \frac{1}{2}$.

Analysis of Distractors
Distractor A is simply the value of the *y*-intercept and represents a misunderstanding of the concept of slope. Distractor B represents an error in the subtraction of $0-(-2)$ or $4-0$. Distractor D represents the error of dividing the change in *x* by the change in *y*.

Strand **Algebra and Functions (AF)**

Standard **7AF3.4**
Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of a line equals the quantities.

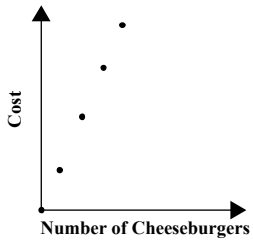
Construct **Conceptual Understanding**

Best Burger sells cheeseburgers for $1.75 each. Part of a table representing the number of cheeseburgers purchased and their cost is shown below.

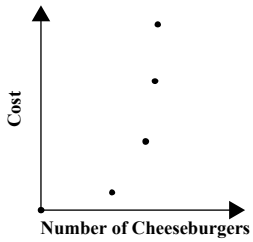
Number Purchased	Cost ($)
0	0
1	1.75
2	3.50
3	5.25
4	7.00

Which of the following is a portion of the graph of the data in the table?

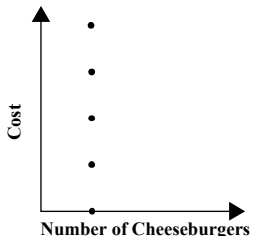
A



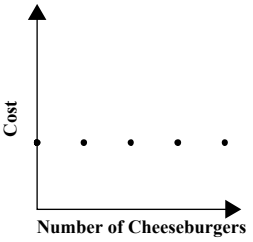
B



C



D



M02112

Graphing direct variation is a powerful way to comprehend and express proportional reasoning. CAHSEE test questions within this standard focus on either of its two main components, both of which require students to understand the relationship between the graphical presentation of data

and the symbolic representation of data. The first component involves the identification of the correct graph. The second component involves the determination of the slope of a direct variation and the interpretation of the meaning of the slope as a constant ratio between the two quantities in the variation.

Sample Test Question
The sample question presents a table of values showing the number of cheeseburgers purchased and the corresponding cost for each additional cheeseburger. Students are asked to select the appropriate graph to represent the data. The correct answer is choice A. Students must understand the relationship between the number of cheeseburgers on the *x*-axis and the corresponding cost for each additional cheeseburger on the *y*-axis. The table shows that each additional cheeseburger purchased increases the total cost by $1.75. In other words, students should recognize that the relationship is linear: As the value on the *x*-axis increases by 1, the value on the *y*-axis increases correspondingly by $1.75.

Analysis of Distractors
The distractors are graphs that incorrectly represent the function in the table. Distractor B shows a nonlinear relationship between number and cost. Cost increases exponentially, so that following the purchase of cheeseburger number three, it has risen higher than the $7.00 indicated in the table. Distractor C is not a function and indicates that there is more than one possible price for the given number of cheeseburgers. Distractor D represents the purchase of additional cheeseburgers without any cost increase beyond the cost of the first cheeseburger.

Strand	**Algebra and Functions (AF)**
Standard	**7AF4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.**
Constructs	**Procedural Skills, Conceptual Understanding, Problem Solving**

In the inequality $2x + \$10{,}000 \geq \$70{,}000$, x represents the salary of an employee in a school district. Which phrase most accurately describes the employee's salary?

A At least $30,000

B At most $30,000

C Less than $30,000

D More than $30,000

M02621

CAHSEE test questions for this standard may focus on any of its components, including solving two-step linear equations, solving two-step inequalities, interpreting the solutions of equations or inequalities, and judging the reasonableness of the solutions of equations or inequalities.

Sample Test Question

The sample test question presents an inequality that represents an employee's salary and asks students to interpret the solution of the inequality in terms of this context. The correct answer is choice A. Students should recognize that the inequality has the solution $x \geq \$30{,}000$ and that the correct way to state this inequality is "at least $30,000."

Analysis of Distractors

The distractors offer incorrect solutions for the inequality and/or ways to express the mathematical notation. Distractor B represents $x \leq \$30{,}000$ which is stated as "at most $30,000." Distractor C represents $x < \$30{,}000$, or "less than $30,000." Distractor D represents $x > \$30{,}000$, or "more than $30,000."

Strand	**Algebra and Functions (AF)**
Standard	**7AF4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.**
Constructs	**Procedural Skills, Conceptual Understanding, Problem Solving**

Stephanie is reading a 456-page book. During the past 7 days she has read 168 pages. If she continues reading at the same rate, how many more days will it take her to complete the book?

A 12

B 14

C 19

D 24

M00380

Problem solving is a significant higher-order thinking skill that enables students to apply their mathematical knowledge to real-world situations. CAHSEE test questions for this standard may require students either to solve a specific problem or to determine the equation that should be used to solve the problem. Test questions may also require students to understand the concept of direct variation and to recognize that direct variation may also be expressed as a linear function. This standard is closely related to Algebra I standard 5.0, which requires students to solve a variety of problems such as rate, work, and percent mixture using algebraic methods.

Sample Test Question

The sample question provides a ratio between pages read and days and asks students to find the number of additional days it will take to read the 456-page book. The correct answer choice is A. One method is to subtract 168 from 456 to obtain the number of pages left. Then set up the proportion $\frac{168}{7} = \frac{288}{x}$ and solve for x.

Analysis of Distractors

The distractors offer solutions that use the values of the problem incorrectly and thus reflect a misunderstanding of the concept. Distractor B represents the number of pages read per day assuming 168 pages is the amount of pages left and can be found by $\frac{168}{12}$. Distractor C represents how many total days are needed to read the book and can be found from the proportion $\frac{168}{7} = \frac{456}{x}$. Distractor D represents the number of pages read per day and can be found by $\frac{456}{19}$.